



# Dynamics of Opinions and Social Power in Complex Social Networks

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## Introduction

Existing models for social dynamics target homogeneous networks:

- Network can have a complex structure.
- Yet, the opinions and links are governed by homogeneous laws.

Existing models are imperfect for studying complex organizations, as

- organizational networks are usually hierarchically structured, and
- at different scales, the networks evolve according to different laws.

Social dynamics in complex networks by example:

- A two-tier organizational networks, where decisions are made *within* and *across* departments or *groups*.
- Each group, discussing an issue, jointly arrives at a consensus decision that best reflects the group's interests.
- Interests | and, hence, decisions | of different groups conflict.
- At the level of the entire organization, the final decision is made based on the groups' decisions, with the group leaders' advocating their respective groups' decisions.
- Leaders are (re-)assessed and their negotiation behavior evolves based on the extent to which the final decision of the organization incorporates the decisions / interests of their respective groups.

## Notation

$W^{(k)}(i) = W^{(k)} + C^{(k)}(i) \in [0; 1]^{n_k \times n_k}$  { adjacency (interpersonal influence) matrix of group  $k$ .  $C^{(k)}(i)$  { dynamic appraisals of the group's leader;  $W^{(k)} = \text{const}$  { other intra-group links.

$x^{(k)}(t; i) \in [0; 1]^{n_k}$  { group  $k$ 's opinions w.r.t. issue  $i$  at time  $t$ .

$x^{(k)}(i) = h!(W^{(k)}(i)); x^{(k)}(0; i) \in \mathbb{R}$  { final opinion of group  $k$

(*group opinion*) w.r.t. issue  $i$  (DeGroot), where  $h!$  { eigencentality.

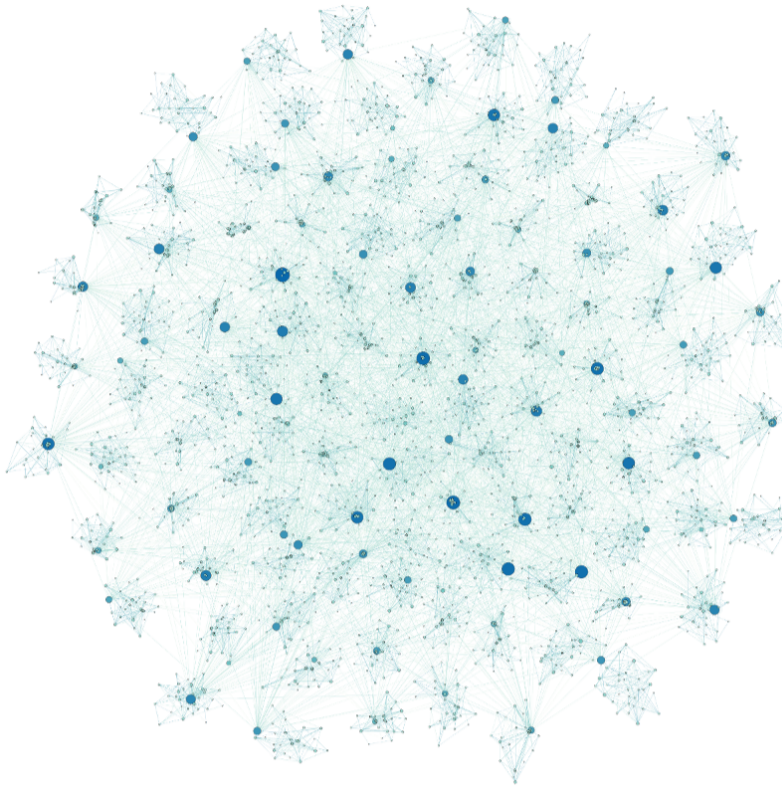
$x^{(L)}(t; i) \in [0; 1]^K$  { opinions of group leaders w.r.t issue  $i$  at time  $t$ .

$x^{(L)}(i) = 1!x^{(L)}(1; i)=K$  { overall final decision (*final opinion*) on issue  $i$  the leaders arrive at.

$W^{(L)}(i) \in [0; 1]^{K \times K}$  { adjacency (interpersonal influence) matrix of the leader network.

$A^{(L)}(i) \in \text{diag}([0; 1])^K$  { leaders' dynamic susceptibilities for the discussion of issue  $i$  among the leaders.

## Conceptual Model



## Questions to Study

How do the opinions evolve in hierarchical networks in a long term?  
What is the long-term evolution of the leaders' social power?  
To what extent does the model reflect real-world decision making?

## Formal Model

- Intra-group discussion of issue  $i$  by each group  $k$ :*

$$x^{(k)}(t+1; i) = W^{(k)}(i)x^{(k)}(t; i); \quad x^{(k)}(i) = h!(W^{(k)}(i)); \quad x^{(k)}(0; i) \in \mathbb{R}$$

- Discussion of issue  $i$  among the leaders:*

$$x^{(L)}(t+1; i) = A^{(L)}(i)W^{(L)}x^{(L)}(t; i) + (I - A^{(L)}(i))x^{(L)}(0; i)$$

$$x^{(L)}(0; i) = x^{(k)}(1; i); \quad x^{(L)}(i) = 1!x^{(L)}(1; i)=K$$

where the leaders' behavior (susceptibility) is defined as

$$A_{kk}^{(L)}(i) = 1 - 1!C_1^{(k)}(i)=n_k$$

(strong leaders can afford to be open to persuasion, while weaker leaders become more protective of their group's opinion to improve their standing inside their respective groups).

- Leaders' "social power" update:*

$$W^{(k)}(i) = W^{(k)} + C^{(k)}(i);$$

$$C^{(k)}(i+1)_{j1} = kC^{(k)}(i)_{j1} + (1 - k)(1 - k)x^{(L)}(i) - x^{(k)}(i)k;$$

where  $k \in [0; 1]$  indicates the extent to which group  $k$ 's leader's reputation is determined by its past value.

## Dynamics of Leaders' Social Power

Leaders' intra-group positions vary; leader network { clique:

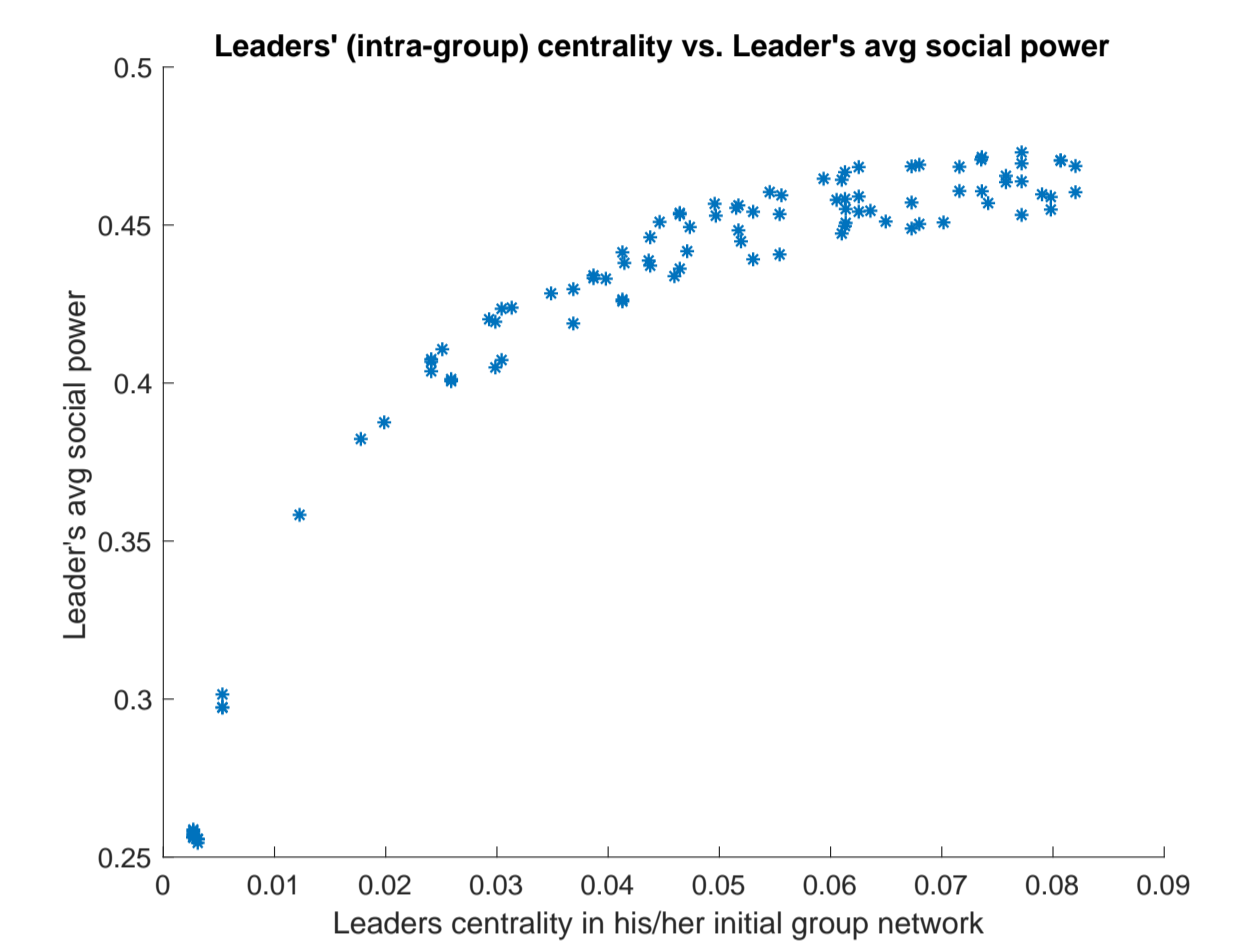


Figure: Leaders' social power

Figure: Leaders' average social power vs. their intra-group position

Leaders' positions in the leader network (but not in groups) vary:

Figure: Leaders' social power

Figure: Leaders' average social power vs. their across-group position

Leaders' positions vary both inside and across the groups:

Figure: Leaders' average social power vs. their intra-group position

Figure: Leaders' average social power vs. their across-group position

Figure: Leaders' average social power vs. their intra- and across-group positions.